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Anisotropic stellar model with class one spacetime and barotropic equation of state

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Abstract

This work presents a realistic stellar model that merged two different approaches in generating a charged anisotropic model. The class one spacetime is used with the Einstein-Maxwell field equations and a barotropic equation of state to investigate various physical properties and behavior of compact stars. The barotropic equation of state p_r=ωρ used to investigate the behavior of compact stellar objects by examining the cosmological constant. The model describes the properties of the phantom dark energy whose cosmological setting is given when ω>1. The barotropic equation of state is equated with the Einstein-Maxwell field equations to obtain the electric field. Then, the class one spacetime is introduced to investigate Einstein-Maxwell field equations. In generating the model, the spacetime manifold was assumed to be flat, static, spherical and symmetric. The gravitational potential z(x) was specified on physical grounds. The chosen metric function z(x) was free from geometric singularities. The physical analysis shows that, metric functions specifically e^v and e^λ exhibit behavior free from geometric singularities and align with expected patterns. Stability criteria as assessed through the adiabatic index are met confirming the model's viability. The study confirms that the model adheres to essential physical criteria including mass profiles, electric fields, compactness factors and charge density.

Keywords: *Anaplasma marginale; Babesia bigemina; Calves; Narok; Risk factors; Seroprevalence*

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Introduction

Modeling of compact stellar objects has become a popular study to examine some of the characteristics including the origin, motion, structure and stability of these objects. The use of equations of state is among the best approach to study these objects. The variability of pressure and energy density found in equations of state affects the physical nature and properties of compact stars. Equations of state are important as they describe the nature of materials composition within compact stars. Hernández *et al.* (2021) generated a model with a barotropic equation of state by generalizing the polytropic equation of state. Lindblom (2010) developed a spectral representation of a cold neutron star with a barotropic equation of state. In the investigation, the necessity of every physical equation of state to satisfy the spectral representation was outlined. A linear equation of state was used by Rej and Bhar (2021) in generating a strange star model in the framework of gravitational theory. Sunzu *et al.* (2019) imposed the linear equation of state in generating a realistic stellar model by specifying the measure of anisotropy and the gravitational potential. Ghosh *et al.* (2022) used the Bayesian scheme to explore the structure and properties of neutron stars at different densities. It was observed that as the density rises from the surface to the interior, different types of matter such as hyperons and kaons were formed. Maharaj *et al.* (2017) and Sunzu and Mashiku (2018) used a quadratic equation of state for a particular measure of anisotropy and a gravitational potential to generate solution to the Einstein-Maxwell field equations for anisotropic matter. Shelote and Wanjari (2021) used quadratic equation of sate to generate a model that examined the relationship between dark energy and dark matter. Lighuda *et al*. (2021) obtained a new class of exact solutions for a three layered astrophysical model with each layer satisfying its own equation of state. Bisht *et al.* (2021) generated well-behaved models for neutron stars with various layers having quark matter by using an equation of state.

Several studies in astrophysics consider pressure anisotropy as an important quantity to be taken into account. Pressure anisotropy can influence several behavior of relativistic compact stars. The interior structure of compact stellar object is influenced by physical characteristics such as pressure, density, mass, and radius. Studies show that as the physical features in the core of stellar bodies may change in density. This causes those bodies to exert both radial pressure and tangential pressure that result from their gravitational pull. However, the work by Sunzu *et al.*, (2019) suggests that a stellar body is considered to be isotropic when the tangential and radial pressures are equal. On the other hand, a stellar body is said to be anisotropic when the two pressures are not equal. Transport coefficients and charge were identified as the origins of pressure anisotropy in star objects, whereby the relativity of particles in anisotropic fluid spheres causes isotropic models to be less successful than anisotropic models. Additionally, it was noted by Rej and Bhar (2021) that anisotropy, as opposed to isotropy, strengthens the stability of stellar objects under radial perturbations. When formulating models with astrophysical significance, charge and anisotropy are thus important factors to be considered.

Higher-dimensional space introduced by the embedding approach has a clear benefit over the initial surface manifold, and that aspect is in its symmetry (Murad, 2018; Singh and Pant, 2016). Braneworld stars have been shown to have non-unique external characteristics caused by radiative-type stress of five-dimensional graviton effects emitted from the core of compact stellar objects (Geddes, 2002). Green *et al.* (2012), Hatefi (2017) and Samanta (2013) described that string theory includes higher dimensional bodies (D-branes) that comprise a single fundamental theory called M-theory that unifies distinct recognized forms of string theories. It has been shown by Govender and Dadhich (2002) that stars gravitational collapse on the brane is followed by Weyl radiation through matching Vaidya solutions and the Reissner-Nordstrom metric. The Vaidya envelope intervention is a unique feature of the collapse of the brane world that is not available in standard four-dimensional space (Maurya and Govender, 2017). Randall and Sundrum (1999) introduced the existence of layers in string theory which is the re-establishment of the ancient concept of Rubakov and Shaposhnikov (1983) that our four-dimensional universe is a four-dimensional surface. The symmetry of four-dimensional spacetime and higher dimensions as addressed by Pavšič (2001) revealed that a 3-braneworld manifold is embedded in a higher-dimensional space. The modified gravity corresponds to modified general relativity which describes the accelerated expansion of the universe resulting in dark energy (Joyce *et al.,* 2016).

Understanding the nature and origin of dark energy is important in quantum gravity, modern cosmology and general theory of relativity. In the late 1990's scientists began to realize that the universe was expanding at an accelerating rate, the concept that revived the existence of dark energy (Joyce *et al.,* 2016). This was done by studying the brightness of distant supernovae-exploding stars. In 2011, Permuter, Schmidt and Riess discovered the cosmic acceleration of the universe due to dark energy, however nobody knows what dark energy actually is (Kolah and Fosmire, 2012). Recent observations show that acceleration is thought to be due to an exotic type of energy called dark energy that stands as a key behind the idea of modifying general relativity (Bhar, 2015; Gupta *et al.,* 2011; Maurya *et al.,* 2023). Embedding of 4D to 5D manifolds have various applications to general relativity, extrinsic gravity, strings and new braneworld. Higher dimensional theories can account for the universe expansion and contractions which save as a model for dark energy. Over years, various possibilities have been explored, with a focus on a 5D solution called the lower energy limit as it represents the simplest space-time extensions to higher dimensional theories (Murad, 2018). For this work, a charged compact star model is investigated by utilizing the Karmarkar condition and a barotropic equation of state. This results to the formulation and analysis of the matter variables. The paper is organized as follows: in the next section we provide the basic Einstein-Maxwell field equations. The embedding condition is explained in Section 3. The exact solutions for the generated model are presented in Section 4. Section 5 provides the analysis of important physical features to be satisfied by a realistic compact star model. The conclusion of the results is outlined in Section 6.

Materials and Methods

Einstein-Maxwell field equations

Two infinitesimal points between invariant distance on the manifold are determined by the line element

$$ds^2 = g_{ij}dx^i dx^j,$$

where g_{ij} represents metric tensor with the standard four-dimensional manifold $x^i = (t, r, \theta, \phi)$. Schwarzschild (1916) introduced the interior line element

$$ds^2 = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2(d\theta^2 + sin^2\,\theta d\phi^2), \tag{1}$$

where λ and ν represent the gravitational potentials. The charged exterior line element as given by the Reissner-Nordström is

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1}dr^2 + r^2(d\theta^2 + sin^2\,\theta d\phi^2), \tag{2}$$

with Q and M representing the mass and charge of the compact star. For charged star, the line elements (1) and (2) are given by

$$\tau^i_j + E^i_j = R^i_j - \frac{1}{2}Rg^i_j, \tag{3}$$

where R^i_j and τ^i_j stand for the Ricci tensor and energy momentum tensors, respectively. R is the Scalar curvature and E^i_j is the electromagnetic field. The electromagnetic field and energy momentum tensor in equation (3) are given by

$$\tau^i_j = (\rho + p_t)\nu^i\nu_j - p_t\delta^i_j + (p_r - p_t)\chi^i\chi_{j,} \tag{4}$$

$$E^i_j = F^{im}F_{jm} - \frac{1}{4}\delta^i_j F^{mn}F_{mn}. \tag{5}$$

In (4), the quantities p_r and p_t stand for the radial and tangential pressures, respectively, ρ is the energy density while ν^i represent a four-velocity vector and χ^i is a unit spacelike vector in the radial direction.

By incorporating the equations (1), (2), (3), (4) and (5), the charged anisotropic Einstein-Maxwell field equations are then given by

$$\rho(r) + \frac{1}{2}E^2 = \frac{1-e^{-\lambda}}{r^2} + \frac{e^{-\lambda}\lambda'}{r}, \tag{6a}$$

$$p_r(r) - \frac{1}{2}E^2 = \frac{\nu' e^{-\lambda}}{r} - \frac{1-e^{-\lambda}}{r^2}, \tag{6b}$$

$$p_t(r) + \frac{1}{2}E^2 = \frac{e^{-\lambda}}{4}\left(2\nu'' + 2\nu'^2 - \nu'\lambda' + \frac{2\nu'}{r} - \frac{2\lambda'}{r}\right), \tag{6c}$$

$$\frac{e^{\frac{\lambda}{2}}}{r^2}(r^2E)' = \sigma, \tag{6d}$$

$$\Delta = p_r - p_t, \tag{6e}$$

where the primes in system (6) represent derivatives with respect to r, Δ stands for the anisotropic factor, and σ is the proper charge density. In this model, the speed of light is considered to be unity $8\pi G = c = 1$.

Field equations via embedding

The Einstein's theory of general relativity has effectively been extended from four to five dimensions. The embedding of dimensions can be extended to even higher dimensions in an effort to create a geometrical unification of all the fundamental interactions. Eisenhart (1966) described the fundamental symmetric tensor $b_{\mu\beta}$ for embedding four-dimensional manifold as

$$R_{\mu\nu\alpha\beta} = \epsilon(b_{\mu\alpha}b_{\nu\beta} - b_{\mu\beta}b_{\nu\alpha}), \qquad (7a)$$

$$0 = b_{\mu\nu;\alpha} - b_{\mu\alpha;\nu}, \qquad (7b)$$

where $\epsilon = \pm 1$ and semicolons stand for covariant differentiation. The line element in equation (1) gives the non-zero values of the fundamental form $b_{\mu\alpha}$. The non-zero values of $R_{\mu\nu\alpha\beta}$ are given by

$$R_{1414} = -e^{\nu}\left(\frac{\nu''}{2} + \frac{\nu'}{4} - \frac{\lambda\prime\nu\prime}{4}\right), \qquad (8a)$$

$$R_{2323} = -e^{\lambda}r^2 sin^2\,\theta\left(e^{\lambda} - 1\right), \qquad (8b)$$

$$R_{1212} = \frac{1}{2}r\lambda', \qquad (8c)$$

$$R_{3434} = -\frac{1}{2}sin^2\,\theta\nu'^{e^{\nu-\lambda}}. \qquad (8d)$$

The corresponding non-zero components of the symmetric tensor $b_{\nu\beta}$ include b_{11}, b_{22}, b_{33}, and b_{14} with $b_{33} = b_{22}sin^2\,\theta$. Substituting these non-zero components into equation (7a) gives

$$R_{1414} = \frac{R_{1212}R_{3434} + R_{1224}R_{1334}}{R_{2323}}. \qquad (9)$$

Equation (9) is the class I spacetime or the Karmarkar condition. For embedding condition, equation (9) must satisfy $R_{2323} \neq 0$ [Govender *et al.,* 2020; Singh *et al.,* 2018].

Using equations (8) and (9) leads to the nonlinear differential equation

$$\frac{\lambda\prime e^{\lambda}}{e^{\lambda}-1} = \frac{2\nu\prime\prime}{\nu\prime} + \nu'. \qquad (10)$$

Integration of equation (10) gives the relation between the metric functions λ and ν as

$$e^{\frac{\nu}{2}} = C + H\int\sqrt{(e^{\lambda(r)} - 1)}\,dr, \qquad (11)$$

where C and H are constants of integration. Equation (11) is used with a barotropic equation of state to form a new exact solution to the Einstein-Maxwell field equations.

By considering transformations similar to that of Durgapal and Bannerji (1983), we introduce new variables as

$$x = r^2, \qquad (12a)$$

$$z(x) = e^{-\lambda}, \qquad (12b)$$

$$y(x) = e^{\nu}. \qquad (12c)$$

These transformations transform system (6) into equivalent forms of the Einstein-Maxwell field equations as

$$\rho(x) + \frac{1}{2}E^2 = \frac{1-z}{x} - 2z', \qquad (13a)$$

$$p_r(x) - \frac{1}{2}E^2 = 2z\frac{y\prime}{y} - \frac{1-z}{x}, \qquad (13b)$$

$$p_t(x) + \frac{1}{2}E^2 = 2xz\frac{y''}{y} + (2z + xz')\frac{y\prime}{y} - xz\frac{y'^2}{y} + z', \qquad (13c)$$

$$\sigma^2(x) = \frac{z}{4\pi x}(xE'^2 + E^2)^2, \qquad (13d)$$

$$\Delta = p_r - p_t. \qquad (13e)$$

Results

In this solution, we present a charged compact star model by merging the barotropic and embedding condition. The approach of merging two different methods in generating new exact solutions to the Einstein-Maxwell field equations has resulted to some compact models with physical significance. Jape *et al.* (2023) utilized the conformal Killing vector to study several properties of a realistic compact star by using a linear equation of state. Mathias *et al.* (2023) employed the class one spacetime to study higher dimensional models of compact stars with a linear equation of state. This work intends to use a barotropic equation of state and class I spacetime to study different properties and behavior of realistic compact stars. The barotropic fluids are the one where pressure is the function of density. The barotropic equation of state relate radial pressure and energy density that use the form

$$p_r = \omega\rho, \qquad (14)$$

where ω is the cosmological constants. Similar forms to the barotropic equation of state (14) have been used by several authors with different approaches other than embedding condition to generate stellar models with physical significances (Hernández *et al.*, 2021; Lindblom, 2010). In Mathias *et al.* (2023) the general linear equation of state was used with negative coefficient condition of the energy density term and embedding condition to develop charged compact star model. Thus, in this study, we have merged the class one spacetime condition (9) and the barotropic equation of state (14) to generate star models that describe the properties of the dark energy whose cosmological setting is given when $\omega > 1$. This approach of merging the class one spacetime and barotropic equation of state with positive values of the cosmological constant to study the properties of charged stars is missing in the existing literature.

Substituting the energy density (13a) and the radial pressure (13b) into the barotropic equation of state (14), the electric field in terms of the metric functions $y(x)$ and $z(x)$ is obtained to be

$$E^2 = \frac{1-z}{x}\left(\frac{2\omega+2}{2+\omega}\right) - \frac{2\omega z\prime}{2+\omega} - \frac{4z}{2+\omega}\frac{y\prime}{y}. \tag{15}$$

It is clear that the electric field (15) can be obtained when the gravitational potentials $y(x)$ and $z(x)$ are known.

To obtain the matter variables, the gravitational potential $z(x)$ is specified on physical grounds to interpret the model's behavior. The potential is used with the class one spacetime condition to get the second gravitational $y(x)$. The embedding (Karmarkar) condition provides the relationship between two gravitational potentials. In this work, the gravitational potential

$$z(x) = \frac{1}{1+bx}, \tag{16}$$

is chosen, where b is a non-zero arbitrary constant. The gravitational potential (16) is continuous, finite and free from geometric singularities. The form (16) is used to generate a realistic stellar model with astrophysical significance. Moreover, there exists some research works that used similar choices to the metric function (16). Linear and quadratic equations of state have been used by Thirukkanesh and Maharaj (2008), and Maharaj and Mafa Takisa (2012), respectively, to generate realistic stellar models with astrophysical significance. For this paper, the gravitational potential (16) is used with the barotropic equation of state with positive cosmological constant and a Karmarkar condition to generate a realistic star model with physical significance.

To obtain the metric function $y(x)$, equation (11) is transformed using transformation (12). This yields

$$y(x) = \left(C + \frac{1}{2}H\int\sqrt{\frac{1-z}{xz}}\,dx\right)^2. \tag{17}$$

Substituting equation (16) into equation (17) gives

$$y(x) = \left(C + \frac{1}{2}Hx\sqrt{b}\right)^2. \tag{18}$$

Having the values of the metric functions in equations (16) and (18), the magnitude of the electric field in equation (15) becomes

$$E^2 = \frac{2\left(4\sqrt{b}H+2bC(1+3\omega)+2b^2C(1+\omega)+b^{\frac{3}{2}}H(5+3\omega)x+b^{\frac{5}{2}}H(1+\omega)x^2\right)}{(1+\omega)(1+bx)^2(2C+Hx\sqrt{b})}. \tag{19}$$

The charged anisotropic stellar model for class I spacetime and the barotropic equation of state is generated by substituting equations (16), (18) and (19) into the system (13). This results to the following matter variables:

$$\rho = \frac{4bC-4\sqrt{b}H+b^{\frac{3}{2}}Hx}{(1+\omega)(1+bx)^2(2C+Hx\sqrt{b})}, \tag{20a}$$

$$p_r = \frac{4bC\omega+2\sqrt{b}H(4(1+bx)+\omega(2+3bx))}{(1+\omega)(1+bx)^2(2C+Hx\sqrt{b})}, \tag{20b}$$

$$p_t = -\frac{\sqrt{b}\left(-4H\omega+4C\sqrt{b}(1+2\omega)+2b^{\frac{3}{2}}C(1+\omega)x+2bH(2+\omega)x+b^2H(1+\omega)x^2\right)}{(1+\omega)(1+bx)^2(2C+Hx\sqrt{b})}, \tag{20c}$$

$$\sigma^2(x) = \left(\frac{-\sqrt{b}CH-4bC^2(1+3\omega)-12b^{\frac{3}{2}}CH(1+\omega)x+2b^{\frac{5}{2}}CH(-1+\omega)x^2+b^3H^2(3+\omega)x^3+b^2x\left(4C^2-3H^2x\right)}{\pi\left(1+\omega\right)^2x(1+bx)^7\left(2C+Hx\sqrt{b}\right)^4}\right)^2. \quad (20d)$$

Discussion

It is important to analyze whether the generated model satisfies several physical requirements for realistic stars. Realistic stellar model used to be free from central singularity and satisfy important conditions including stability, causality, equilibrium, energy behavior and the behavior of mass-radius relationship and the surface redshift. In model formulation, the Python programming language was used to obtain the profiles for the gravitational potentials and matter variables with the constants: $b = 0.01,\ H = 0.0001585, \omega = 1.001, C = 0.01.$ The parameters are specified so that a well-behaved model is generated.

Regularity

The regularity condition requires the metric functions $y(x)$ and $z(x)$ to be continuous throughout the stellar interior. At the centre of the star, the gravitational $z(x)$ needs to satisfy the condition $z(x) = e^{\lambda(x=0)} = 1$, while the gravitational potential $y(x)$ must be greater than zero, that is, $y(x) = e^{\nu(x=0)} > 0$. These conditions are satisfied by our model as indicated in **Figure 1**. Similar trends are observed in models generated by Mathias *et al.,* (2023) and Sharma *et al.,* (2021). The equations for the metric function $z(x)$ and $y(x)$ are given in (16) and (18). The presence of anisotropic pressure is also used to determine the regularity condition. Radial and tangential pressures vary in this model where the pressures are monotonically decreasing functions with maximum values at the centre. The energy density ρ must be positive, finite within the interior structure, and decreasing function. The plots for energy density, radial and tangential pressures are given in **Figure 2.** These plots are in agreement with those generated by Habsi *et al.,* (2023) and Upreti *et al.,* (2020).

Energy conditions

Any physically realistic model should satisfy the energy conditions. The analysis for the null, weak, strong, and the dominant energy conditions is clearly presented. This is done by using the following inequalities:

$$\text{NEC:} \quad \rho + \frac{E^2}{2} \geq 0, \quad (21)$$

$$\text{WEC:} \quad \rho - p_t \geq 0,\ \ \rho - p_r + E^2 \geq 0, \quad (22)$$

$$\text{SEC:} \quad \rho - 2p_t - p_r + \frac{E^2}{2} \geq 0, \rho - 3p_r + E^2 \geq 0,\ \ \rho - 3p_t \geq 0, \quad (23)$$

$$\text{DEC:} \quad \rho - |p_t| \geq 0,\ \ \rho - |p_r| + E^2 \geq 0. \quad (24)$$

(Bhattacharjee *et al.,* 2024; Maurya *et al.,* 2018; Pant *et al.,* 2022; Lighuda *et al.,* 2022). The model generated in this work satisfies all these conditions as indicated in **Figure 3.**

Stability through adiabatic index and causality condition

A relativistic stellar model should satisfy the stability through the adiabatic index (Γ) where its value is required to be greater than $\frac{4}{3}$. The stability for the charged anisotropic model is determined by the formula

$$\Gamma = \frac{(\rho+p_r)}{p_r}\frac{dp_r}{d\rho}, \quad (24)$$

(Mathias *et al.,* 2022; Sunzu and Mathias, 2022). Moreover, in analyzing stability through causality condition, the speed of sound inside the stellar interior is required to be less than the speed of light (Lighuda *et al.,* 2021). The formula for the radial and tangential speeds are given by

$${v_r}^2 = \frac{dp_r}{d\rho},\ \ {v_t}^2 = \frac{dp_t}{d\rho}, \quad (25)$$

(Gedela and Bisht, 2023; Upret *et al.,* 2023). **Figure 4** shows that the value of adiabatic index obtained in this model is in the required acceptable range. Similar structures are also observed in the work of Maharaj and Mafa Takisa (2012) and Lighuda *et al.,* (2022). The values for the radial and the tangential speed of sound were found to be in the range $0 < {v_r}^2, {v_t}^2 < 1$ as required.

Equilibrium condition

For the equilibrium condition, the total forces acting within the star should balance. The Tolman-Oppenheimer-Volkoff (TOV) equation in the presence of an electric field is used to examine the equilibrium condition. It is given by equation

$$-\frac{M_G(\rho+p_r)}{r^2}e^{\frac{\lambda-\nu}{2}}-\frac{dp_r}{dr}+\sigma E^2 e^{\frac{\lambda}{2}}+\frac{2\Delta}{r}=0, \qquad (26)$$

where M_G represents the effective gravitational mass with

$$M_G(r)=\frac{1}{2}r^2\nu' e^{\frac{\nu-\lambda}{2}}. \qquad (27)$$

Substituting equation (27) into (26) gives

$$-\frac{\nu'}{2}(\rho+p_r)-\frac{dp_r}{dr}+\sigma E^2 e^{\frac{\lambda}{2}}+\frac{2\Delta}{r}=0. \qquad (28)$$

Equation (28) contains four different forces. These forces include the anisotropic, gravitational, hydrostatic, and electric, and are given as

$$F_a=\frac{2\Delta}{r}, \qquad (29a)$$

$$F_g=-\frac{\nu'}{2}(\rho+p_r), \qquad (29b)$$

$$F_h=-\frac{dp_r}{dr}, \qquad (29c)$$

$$F_e=\sigma E^2 e^{\frac{\lambda}{2}}, \qquad (29d)$$

respectively. If the sum of the forces in system (29) is zero within the stellar interior, that is,

$$F_a+F_g+F_h+F_e=0,$$

then the star model is stable. The transformations of the forces F_a, F_g, F_h and F_e give the following equivalent forms:

$$F_a=x^{\frac{1}{2}}z\dot{y}(\rho+p_r(x)), \qquad (30a)$$

$$F_h=-\frac{dp_r}{dx}, \qquad (30b)$$

$$F_e=\frac{zE^2(x\dot{E}^2+E^2)}{3.544x^{\frac{1}{2}}}, \qquad (30c)$$

$$F_e=\frac{\Delta}{r^{\frac{1}{2}}}. \qquad (30d)$$

The sum of forces in system (30) sums up to zero for equilibrium as indicated in **Figure 5,** this shows that the model generated is realistic and well behaved.

Mass-radius condition

In this model, the mass function is obtained by relating the equations for the line elements (1) and (2). This is done by equating the gravitational potential λ as

$$e^{\lambda}=\frac{1}{1-\frac{2M}{r}+E^2}. \qquad (31)$$

(Gade and Sharma, 2022; Sunzu and Lighuda, 2023). Incorporating equations (16) and (19) into (31), the mass function is obtained as

$$M=\frac{\sqrt{x}\left(8\sqrt{bH}+2b^2C(1+\omega)x(2+x)+b^{\frac{5}{2}}H(1+\omega)x^2(2+x)+2bC\left(2+x+\omega(6+x)\right)+b^{\frac{3}{2}}Hx(10+x+w(6+x))\right)}{2(1+\omega)(1+bx)^2(2C+\sqrt{b}Hx)}. \qquad (32)$$

The compactness factor is analyzed by using the formula

$$\mu=\frac{2M}{r}, \qquad (33)$$

(Maurya *et al.,* 2019; Matondo and Maharaj, 2021). Utilizing the mass function (32), the compactness factor in equation (33) becomes

$$\mu=\frac{16\sqrt{b}CH+4b^{\frac{7}{2}}CH(1+\omega)(-2+x)x^3+b^4H^2(1+\omega)(-2+x)x^4+16b^{\frac{5}{2}}CHx^2+(-1+\omega(-3+x)+b^{\frac{3}{2}}CHx(2+3x)}{4(1+\omega)\sqrt{x}(1+bx)^3(2C+\sqrt{b}Hx)^2}$$

$$+\ \frac{3\omega(2+x))4b^3x^2\left(C^2(1+\omega)(-2+x)+H^2x+3H^2x\left(-10+x+\omega(2+x)\right)\right)+4b(-2H^2x(C^2(2+x)))}{4(1+\omega)\sqrt{x}(1+bx)^3(2C+\sqrt{b}Hx)^2}. \qquad (34)$$

The maximum value of the compactness factor is found to be 1.9 as indicated in **Figure 6.** This value is within the range as required in the literature. The plots for mass function and compactness factor are well behaved.

Anisotropic factor

The anisotropic factor is examined when tangential pressure is not equal to radial pressure. This is represented by the symbol Δ and given by

$$\Delta=p_r-p_t, \qquad (35)$$

(Bhar, 2023; Bhar *et al.,* 2017; Maurya and Govender, 2017). The anisotropic factor equation (35) can be easily obtained by substituting equations (20b) and (20c) for the tangential and radial pressures to give

$$\Delta = \frac{8\sqrt{b}H+4b(C+3Cw)+2b^2C(1+\omega)x+4b^{\frac{3}{2}}H(3+2\omega)+b^{\frac{5}{2}}H(1+\omega)x^2}{(1+\omega)(1+bx)^2(2C+\sqrt{b}Hx)}. \quad (36)$$

Electric field

A charged particle is a source of force in astrophysics. Strong nuclear force, electromagnetic force, and gravitational force are some examples of these forces. The balance between the number of protons and electrons in an atom is frequently used to determine the charged particle. The charge must have zero magnitude at the centre and attains its highest value towards the boundary of the relativistic star (Maurya *et al.,* 2022; Ratanpal and Patel, 2023; Sharma and Ratanpal, 2013; Mathias *et al.,* 2021; Maharaj and Brassel, 2021). Equation for the electric field is given in (19).

Conclusion

In this study, we have generated a charged anisotropic star model admitting an embedding condition and a barotropic equation of state. The barotropic equation of state with the Einstein-Maxwell equations are used to obtain an equation for the electric field intensity. The gravitational potential $z(x)$ was specified on physical grounds to obtain the second gravitational potential $y(x)$ from Karmarkar condition. This process enabled to get all matter variables for analysis. A thorough analysis of the physical relativistic conditions was conducted to determine whether the barotropic model is realistic. It was found that the gravitational potentials are free from physical and geometric singularities. The regularity condition for the behavior of the energy density (ρ) radial and tangential pressures p_r and p_t are satisfied. The model is also isotropic at the centre. The adiabatic stability condition and causality criterion are satisfied as well. The total physical forces found to balance for equilibrium requirements, and all the energy conditions are also satisfied. It is also noted that the compactness factor values are within the acceptable ranges for relativistic charged anisotropic compact stars.

Recommendations

The analysis of the model indicates that merging of the embedding condition with a barotropic equation of state leads to a new realistic compact star models with physical significance. Based on the results obtained in this model, other researchers may consider different forms of equations of state like Van der Waals, polytropic or quadratic and other forms of gravitational potentials with embedding condition. This may result to new classes of exact solutions to the Einstein-Maxwell equations.

Figure 1

Metric functions e^{λ} *and* e^{ν} *versus radial interval R*

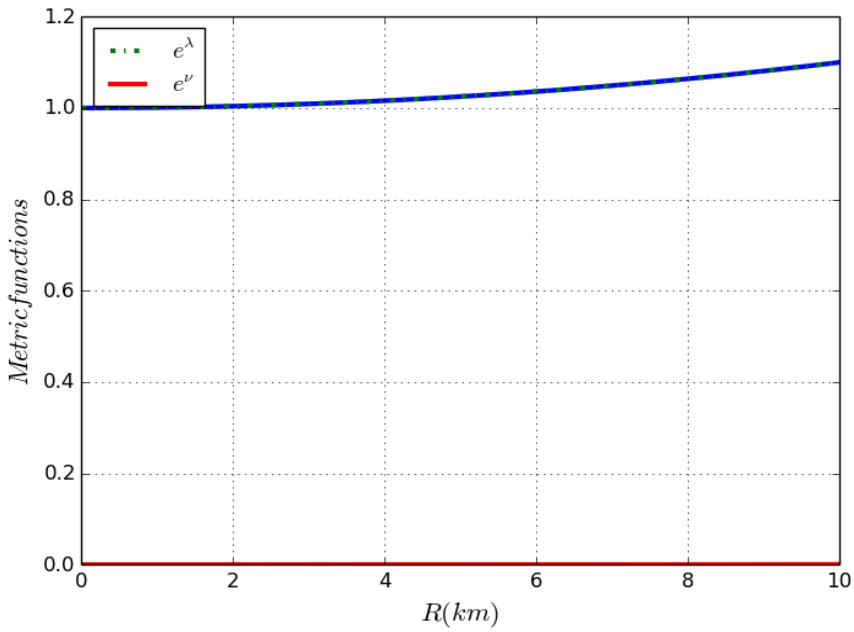


Figure 2

Matter variables ρ, p_r *and* p_t *versus radial interval R*

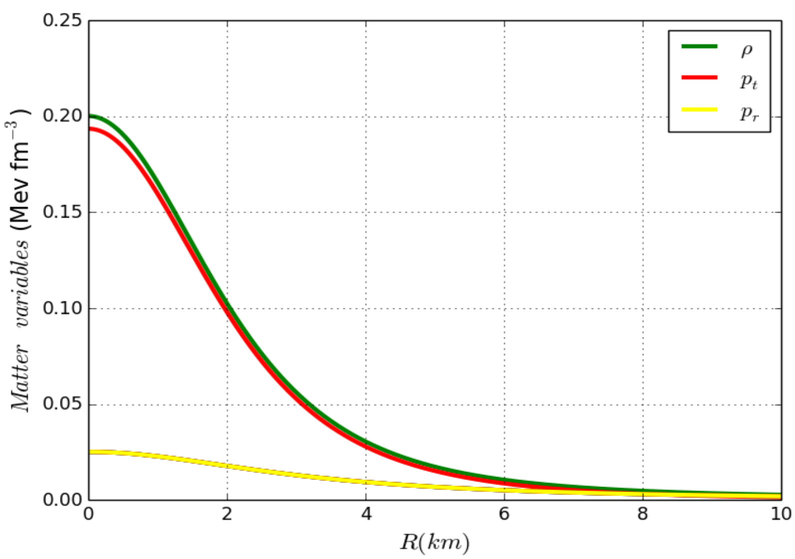


Figure 3

Energy conditions versus radial interval R

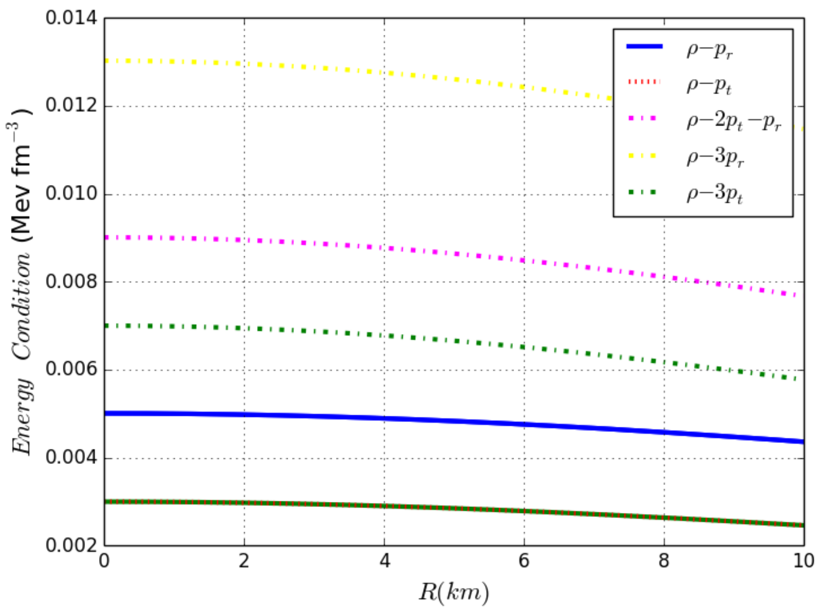


Figure 4

Stability Γ versus radial interval R

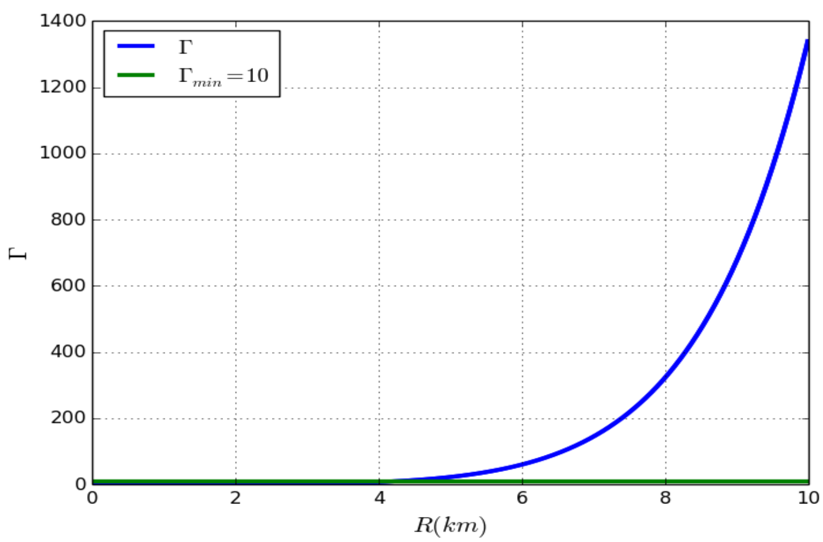


Figure 5

Forces versus radial interval R

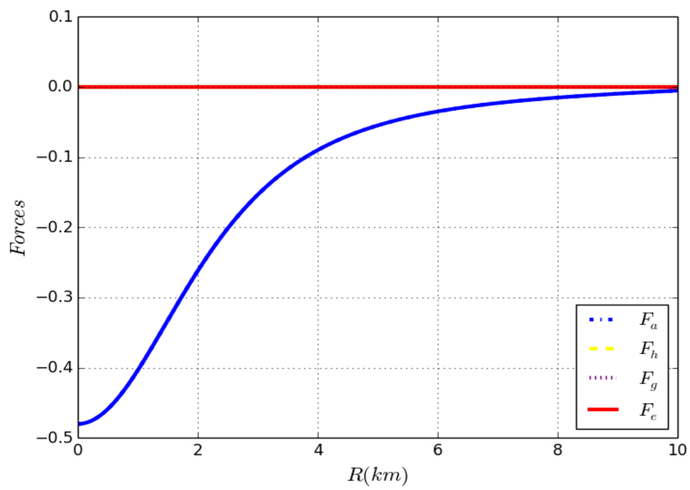
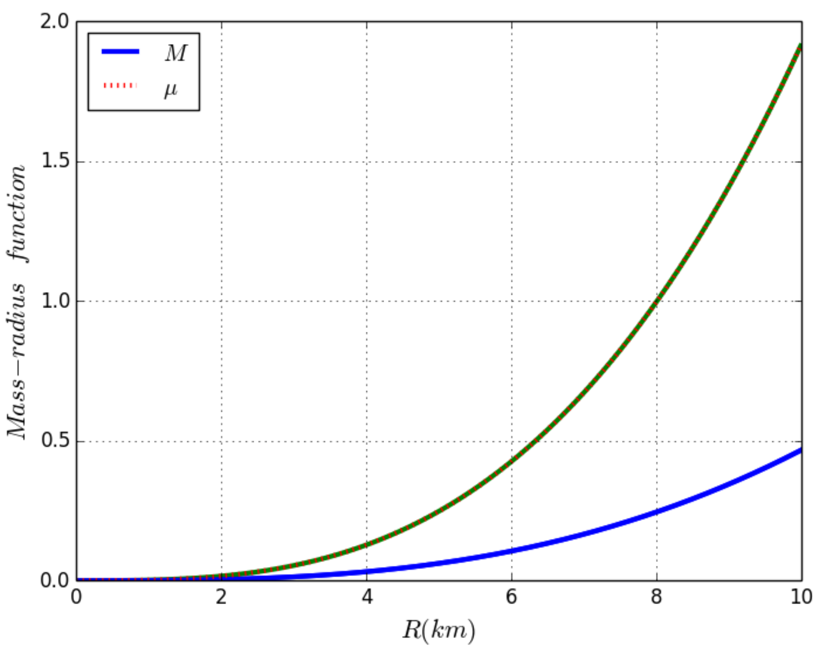


Figure 6

Mass-radius function versus radial interval R



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